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ONE EXAMPLE OF MAGNETOHYDRODYNAMIC STOKES FLOW AROUND A SELF-PROPELLED SPHERE

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Magnetohydrodynamic flow around a sphere equipped with an internal source of electromagnetic fields in the form of a variable magnetic dipole is investigated in the Stokes approximation. This dipole is capable of imparting translational motion to the sphere relative to the liquid. The properties of streamline flow in the self-propelled mode of operation of the source due to the influence of distributed volumetric forces on the character of the flow are demonstrated.

The creation of magnetohydrodynamic (MHD) motors for seawater and of the means for controlling the flow pattern using volumetric electromagnetic forces (VEFs) has now become a practical task [1]. The corresponding problems of magnetohydrodynamic flow around bodies have long attracted the interest of researchers. The closed problem of MHD flow around a self-propelled body with an internal source of fields was first solved by Khonichev and Yakovlev [2]. The steady Stokes motion of a sphere caused by a variable magnetic dipole displaced from the center of the sphere was investigated for the limiting case of a strong skin effect. The study of this system was continued and new results were obtained that were not published at that time. They involve the influence of the parameters of the source on the translational velocity of the sphere and the size of the separation zone; one of the unexpected results is related to the possibility of the sphere moving in the opposite direction from that of the force exerted on the dipole by the magnetic field of currents in the liquid.

These results are more curious than practical, but they are of scientific interest, since they demonstrate the capability of distributed VEFs for altering the hydrodynamic flow pattern.

1. In the present work, we consider the self-propelled mode of flow over a sphere in a conducting liquid. The sphere is equipped with an electromagnetic source, for which we take a variable magnetic dipole displaced from the center of the sphere and oriented as shown in Fig. 1. The sphere is assumed to be nonconducting and nonmagnetic, its radius is a, the conductivity of the liquid is σ, and the frequency of variation of the magnetic moment is ω.

A qualitative explanation of why a sphere with such an internal source can be self-propelled consists in the following. The variable magnetic dipole under consideration produces in the surrounding liquid a vortical electric field and vortical currents having only an azimuthal α component. The force $\mathbf{F} = (-F_0 + F e^{2i\omega t})\,\mathbf{e}_z$ exerted by the magnetic field of these variable circular currents on the magnetic dipole consists of two parts — constant and variable. The constant component, which differs from zero when the dipole is displaced from the center of the sphere, is an attractive electromagnetic force exerted on the sphere. It cannot be balanced by a pressure gradient in the liquid, since the VEFs $\mathbf{f} = (1/c)[\mathbf{j} \times \mathbf{H}]$ generated in the liquid are not potential forces in general, so that the force $-F_0\mathbf{e}_z$ produces translational motion of the sphere relative to the liquid.

The problem is solved in the Stokes approximation under additional assumptions that provide for splitting of the general MHD problem into purely electrodynamic and hydrodynamic parts. This is possible

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if the second term in Ohm's law $\mathbf{j} = \sigma\{\mathbf{E} + (1/c)[\mathbf{v} \times \mathbf{H}]\}$, is negligibly small compared to the first:

$$(1/c)|[\mathbf{v} \times \mathbf{H}]| \ll |\mathbf{E}|, \tag{1.1}$$

i.e., the current density does not depend on the velocity field. In this case, the $\mathbf{E}$ and $\mathbf{H}$ fields, as well as the field of volumetric forces f, do not depend on the velocity field, and the electrodynamic part of the problem can be separated from the hydrodynamic part. Since the vortical electric field is proportional to ω, condition (1.1) is satisfied at sufficiently high frequencies:

$$\omega \gg (v_0/a). \tag{1.2}$$

Here v_0 is the velocity scale, determined in the course of the solution.

2. The electrodynamic problem of a variable magnetic dipole in a spherical cavity within an unbounded conducting space that arises here is solved using the vector potential $\mathbf{A} = H_0 a A(r,\theta) e^{i\omega t} \mathbf{e}_\alpha$, where $H_0 = m_0/a^3$. The dimensionless function $A(r,\theta)$ satisfies the equation

$$\operatorname{curl}\operatorname{curl}[A(r,\theta)\mathbf{e}_\alpha] = \begin{cases} -(2i/\delta^2)A(r,\theta)\mathbf{e}_\alpha & \text{for } r > 1, \\ 0 & \text{for } r < 1 \end{cases} \tag{2.1}$$

($\delta = c/\sqrt{2\pi\sigma\omega}a$ is the dimensionless thickness of the skin layer), the boundary conditions of continuity of the function A and its derivative $\partial A/\partial r$ at the surface of the sphere ($r = 1$), and the condition of boundedness at infinity. The solutions inside (A_1) and outside (A_2) the sphere have the form

$$A_1 = A_m(r,\theta) + \sum_{l=1}^{\infty} c_l r^l P_l^1(\cos\theta), \quad A_2 = \sum_{l=1}^{\infty} b_l \frac{1}{\sqrt{r}} H_{l+1/2}^2(sr) P_l^1(\cos\theta). \tag{2.2}$$

Here $s = (1-i)/\delta$, $P_l^1(\cos\theta) = -\sin\theta(d/d\cos\theta)[P_l(\cos\theta)]$ and $H_{l+1/2}$ are associated Legendre functions and secondary Hankel functions of half-integral order, and $A_m(r,\theta) = r\sin\theta/(\varepsilon^2 + r^2 - 2r\varepsilon\cos\theta)^{3/2}$ is a function that gives the vector potential of the magnetic dipole. The latter can also be expanded in a series in $P_l^1(\cos\theta)$ with coefficients in the form of powers of ε/r, where $\varepsilon = d/a$ is the relative displacement of the dipole from the center of the sphere,

$$A_m = -(1/r^2)\sum_{l=1}^{\infty}(\varepsilon/r)^{l-1} P_l^1(\cos\theta).$$

This makes it possible, using the boundary conditions at the surface ($r = 1$), to determine the coefficients b_l and c_l from (2.2):

$$b_l = \frac{2l+1}{sH_{l+1+1/2}^{(2)}(s)}\varepsilon^{l-1}, \qquad c_l = -\frac{H_{l-1/2}^{(2)}(s)}{H_{l+1+1/2}^{(2)}(s)}\varepsilon^{l-1}.$$

Thus the solution of the electrodynamic problem is completed.

Hence, the unknown force exerted on the dipole by the magnetic field of currents in the liquid is determined by the gradient of this field at the location of the dipole. The result for the time-averaged quantity F_z has the form

$$\langle F_z \rangle = -F_0 = -H_0^2 a^2 F_1(\varepsilon,\delta), \quad F_1(\varepsilon,\delta) = -\mathrm{Real}\sum_{l=2}^{\infty}(l-1)c_l\varepsilon^{l-2}\left.\frac{P_l^1(\cos\theta)}{\sin\theta}\right|_{\theta=0}. \tag{2.3}$$

The volumetric electromagnetic forces and the curl of these forces are calculated independently of the velocity field as $\mathbf{f} = (\sigma/c)[\mathbf{E} \times \mathbf{H}]$. The result reduces to the form

$$f_r = f_0\frac{1}{r}\mathrm{Real}\left[iA^*\frac{\partial}{\partial r}(rA) - iA\frac{\partial}{\partial r}(rA)e^{2i\omega t}\right],$$

$$f_\theta = f_0\frac{1}{r\sin\theta}\mathrm{Real}\left[iA^*\frac{\partial}{\partial\theta}(A\sin\theta) - iA\frac{\partial}{\partial\theta}(A\sin\theta)e^{2i\omega t}\right], \quad f_\alpha = 0,$$

$$\operatorname{curl}\mathbf{f} = \frac{f_0}{a}\,[\Phi_0(r,\theta) + \mathrm{Real}\tilde{\Phi}(r,\theta)e^{2i\omega t}]\mathbf{e}_\alpha,$$

$$\Phi_0(r,\theta) = \frac{2}{r}\,\mathrm{Real}\Big(i\,\frac{\partial A}{\partial\theta}\,\frac{\partial A^*}{\partial r}\Big),\quad \tilde{\Phi}(r,\theta) = \frac{2}{r}\,iA\Big(\frac{1}{r}\,\frac{\partial A}{\partial\theta} - \cot\theta\,\frac{\partial A}{\partial r}\Big),\quad f_0 = \frac{\sigma\omega H_0^2 a}{2c^2}.$$

(Here and below, complex-conjugate quantities are marked by asterisks.) It is seen from these equations that the force $\mathbf{f}$ and the curl of forces, curl $\mathbf{f}$, have both a stationary and an oscillating (with a frequency 2ω) component, and they are of the same order of magnitude. Because of this, the investigated flow also comprises analogous parts.

3. The hydrodynamic part of the problem reduces to the solution of the equations

$$\operatorname{div}\mathbf{V} = 0,\quad \operatorname{curl}\mathbf{V} = \mathbf{W},\quad \frac{\partial\mathbf{W}}{\partial t} + \nu\operatorname{curl}\operatorname{curl}\mathbf{W} = \frac{1}{\rho}\operatorname{curl}\mathbf{f},$$

which contain the curl of the electromagnetic forces $\mathbf{f}$. In connection with the linearity of these equations, the stationary and oscillating components in the velocity field are determined independently of each other. At the high frequencies under consideration (1.2), the oscillating component is small compared to the stationary component (it is not given here).

The problem for the stationary flow component comes down to determining the dimensionless stream function $\psi(r,\theta)$ and the vorticity $w(r,\theta)$, introduced using a certain velocity scale $\tilde{v}_0$:

$$\mathbf{V} = \tilde{v}_0\operatorname{curl}[\psi(r,\theta)\mathbf{e}_\alpha],\qquad \mathbf{W} = \frac{\tilde{v}_0}{a}\,w(r,\theta)\mathbf{e}_\alpha.$$

The functions ψ and w satisfy the equations

$$\operatorname{curl}\operatorname{curl}[w(r,\theta)\mathbf{e}_\alpha] = \frac{2f_0a^2}{\rho\nu\tilde{v}_0}\,\frac{1}{r}\,\mathrm{Real}\Big[i\,\frac{\partial A_2}{\partial\theta}\,\frac{\partial A_2^*}{\partial r}\Big]\mathbf{e}_\alpha; \tag{3.1}$$

$$\operatorname{curl}\operatorname{curl}[\psi(r,\theta)\mathbf{e}_\alpha] = w(r,\theta)\mathbf{e}_\alpha \tag{3.2}$$

and the boundary conditions

$$\begin{aligned} &w = 0, && \psi = (1/2)(u_0/\tilde{v}_0)r\sin\theta = -(1/2)(u_0/\tilde{v}_0)rP_1^1(\cos\theta) \quad \text{for } r\to\infty,\\ &\psi = 0, && \frac{\partial\psi}{\partial r} = 0 \quad \text{for } r = 1. \end{aligned} \tag{3.3}$$

The dimensionless complex on the right side of Eq. (3.1) represents the ratio of the scales of the electromagnetic and viscous forces, i.e., the square of the Hartmann number. We have already emphasized that the velocity scale of the stated problem is undefined. It seems intuitively true that the characteristic velocity of the flow due to the applied electromagnetic forces is such that the viscous forces are balanced, to order of magnitude, by the volumetric force, and, hence, the aforementioned dimensionless complex equals unity. From this condition we determine the quantity

$$\tilde{v}_0 = \frac{\sigma\omega H_0^2 a}{\rho c^2\nu} = \frac{H_0^2 a}{2\pi\rho\nu}\,\frac{1}{\delta^2}, \tag{3.4}$$

which is used temporarily as the velocity scale.

In conditions (3.3), u_0 is the velocity of the oncoming stream (in the coordinate system of the sphere). Since in the problem of a self-propelled sphere under consideration, the velocity of the latter is unknown, boundary conditions (3.3) must be supplemented by the equation of motion of the sphere, which in the case of steady motion reduces to the equality

$$\langle F_z\rangle + T_z = 0, \tag{3.5}$$

where $\langle F_z\rangle$ is defined in (2.3), $T_z = 2\pi a^2\int_0^\pi(\sigma_{rr}\cos\theta - \sigma_{r\theta}\sin\theta)\sin\theta\,d\theta$ is the projection of the resultant viscous stresses $\sigma_{rr} = -p(1,\theta)$ and $\sigma_{r,\theta} = \rho\nu(\tilde{v}_0/a)w(1,\theta)$ applied to the surface of the sphere by the liquid.

The pressure distribution over the surface of the sphere is found from the equation of motion of the liquid and is expressed in terms of $\langle f_\theta \rangle$ and the quantity $(\tilde{v}_0/a)\rho\nu(\partial/\partial r)(rw)\big|_{r=1}$. The shear stresses are determined by the vorticity at the surface of the sphere. The result of the calculations for T_z has the form

$$T_z = 2\pi a^2 \left\{ \frac{a}{2} \int_0^\pi \langle f_\theta(1,\theta) \rangle \sin^2(\theta) d\theta + \frac{1}{2} \frac{\tilde{v}_0}{a} \rho\nu \int_0^\pi \left[\frac{\partial w}{\partial r}\Big|_{r=1} - w(1,\theta) \right] \sin^2(\theta) d\theta \right\}. \tag{3.6}$$

The differential operators in Eqs. (3.1) and (3.2) coincide with the operator in Eq. (2.1). The solutions for ψ and w can therefore be constructed by separating the variables if the function

$$\mathrm{Real}\left(i \frac{\partial A_2}{\partial \theta} \frac{\partial A_2^*}{\partial r} \right) = \mathrm{Real}\left\{ i \sum_{l=1}^{\infty} \sum_{l'=1}^{\infty} \frac{b_l}{\sqrt{r}} H_{l+1/2}^{(2)}(sr) \frac{d}{d\theta} [P_l^1(\cos\theta)] b_{l'}^* \frac{d}{dr} \left[\frac{1}{\sqrt{r}} H_{l'+1/2}^{(2)}(s^* r) \right] P_{l'}^1(\cos\theta) \right\},$$

on the right side of (3.1) is represented as an expansion in associated Legendre functions $P_l^1(\cos\theta)$. One can show that the product $P_l^1(\cos\theta)(d/d\theta)P_{l'}^1(\cos\theta)$ is represented as a finite sum $\sum_{n=0}^{M} C_{l,l',n} P_{l+l'-2n}^1(\cos\theta)$, where

$$M = \begin{cases} (l+l')/2 - 1, & \text{if} \quad l+l' \text{ is an even number,} \\ (l+l'-1)/2, & \text{if} \quad l+l' \text{ is an odd number.} \end{cases}$$

Solutions of Eqs. (3.1) and (3.2) in the general case (2.2) are therefore obtained in the form of double series. The awkwardness of these solutions hinders their physical analysis, nullifying the advantages of the analytical solution over a numerical solution. Therefore, here we investigate the relatively simple case

$$\varepsilon = d/a \ll 1 \tag{3.7}$$

of a small displacement of the dipole from the center of the sphere, in which we can be confined to a small number of series terms in the solution. At the same time, this solution is not trivial; it demonstrates interesting properties of the flow around a sphere set in motion by an internal source of fields, and it lets us give them a physical explanation.

In satisfying condition (3.7), we retain only the terms of zeroth and first order with respect to the parameter ε in the expressions for the field of forces **f** and the thrust **F**. Then, the right side of (3.1) is easily reduced to the required form: the sum over the functions $P_l^1(\cos\theta)$:

$$\frac{1}{r} \mathrm{Real}\left(i \frac{\partial A_2}{\partial \theta} \frac{\partial A_2^*}{\partial r} \right) = -\frac{1}{r} \sum_{l=1}^{3} \Phi_l(r) P_l^1(\cos\theta),$$

$$\Phi_1(r) = -\varepsilon \frac{9\delta^2}{2} \mathrm{Real} \frac{i}{H_{5/2}^*(s) H_{7/2}(s)} \frac{1}{\sqrt{r}} \left[3H_{5/2}(sr) \frac{d}{dr} \left(\frac{1}{\sqrt{r}} H_{3/2}^*(sr) \right) + H_{3/2}^*(sr) \frac{d}{dr} \left(\frac{1}{\sqrt{r}} H_{5/2}(sr) \right) \right], \tag{3.8}$$

$$\Phi_2(r) = \frac{3\delta^2}{2} \frac{1}{|H_{5/2}(s)|^2} \frac{1}{r} \mathrm{Real}\, i s^* H_{3/2}(sr) H_{1/2}^*(sr),$$

$$\Phi_3(r) = \varepsilon 3\delta^2 \mathrm{Real} \frac{i}{H_{5/2}(s) H_{7/2}^*(s)} \frac{1}{\sqrt{r}} \left[H_{3/2}(sr) \frac{d}{dr} \left(\frac{1}{\sqrt{r}} H_{5/2}^*(sr) \right) - 2H_{5/2}^*(sr) \frac{d}{dr} \left(\frac{1}{\sqrt{r}} H_{3/2}(sr) \right) \right].$$

(Here H_ν are second Hankel functions $H_\nu^{(2)}$.) Series (2.3) for the dimensionless thrust begins with a term proportional to ε, and in the approximation under consideration we have

$$F_1(\varepsilon,\delta) = -3\varepsilon \,\mathrm{Real} \frac{H_{3/2}(s)}{H_{7/2}(s)} = 6\varepsilon \frac{4 + 14\delta + 12\delta^2}{(2 + 12\delta + 15\delta^2)^2 + [2 - 15\delta^2(1+\delta)]^2}. \tag{3.9}$$

The solution of Eq. (3.1) with the right side (3.8) bounded at infinity has the form

$$w(r,\theta) = \sum_{l=1}^{3} \left\{ \alpha_l r^{-(l+1)} + \frac{1}{2l+1} \int_r^\infty \left[\left(\frac{x}{r}\right)^{l+1} - \left(\frac{x}{r}\right)^{-l} \right] \Phi_l(x)\, dx \right\} P_l^1(\cos\theta), \tag{3.10}$$

and Eq. (3.2) has the solution

$$\psi(r,\theta)=\sum_{l=1}^{3}\psi_l(r)P_l^1(\cos\theta),$$

$$\psi_l(r)=\mu_l r^{-(l+1)}+\beta_l r^l+\frac{\alpha_l}{2(2l-1)}r^{-(l-1)}+\frac{1}{2l+1}\gamma_l(r), \tag{3.11}$$

$$\gamma_l(r)=-\frac{1}{2}\int_r^\infty x^2\Phi_l(x)\left[\frac{1}{2l+3}\left(\left(\frac{x}{r}\right)^{l+1}-\left(\frac{x}{r}\right)^{-(l+2)}\right)-\frac{1}{2l-1}\left(\left(\frac{x}{r}\right)^{l-1}-\left(\frac{x}{r}\right)^{-l}\right)\right]dx.$$

The constants α_l, β_l, and μ_l are determined from boundary conditions (3.3) and Eq. (3.5). From the condition at infinity we get the equations $\beta_1=-u_0/2\tilde{v}_0$, $\beta_2=0$, and $\beta_3=0$. To use (3.5), we first note that the first integral in (3.6) is proportional to ε, since $\langle f_\theta\rangle$ differs from zero only for a dipole displaced from the center of the sphere:

$$\langle f_\theta(r,\theta)\rangle=-\varepsilon\,\frac{45\delta^2}{2}\,f_0\sin^3\theta\,\frac{1}{r^2}\,\mathrm{Real}\,\frac{iH^*_{3/2}(sr)H_{5/2}(sr)}{H^*_{5/2}(s)H_{7/2}(s)}.$$

The first term from the sum (3.9), also due to the displacement of the dipole from the center of the sphere, makes a nonzero contribution to the second integral of (3.6). We finally have

$$T_z=2\pi a^2\left\{\frac{3}{\pi}\,\varepsilon\,H_0^2\mathrm{Real}\,\frac{iH_{3/2}(s)H_{5/2}(s)}{H^*_{7/2}(s)H_{5/2}(s)}+\frac{2}{3}\frac{\tilde{v}_0}{a}\,\rho\nu\left[3\alpha_1+\int_1^\infty x^2\Phi_1(x)\,dx\right]\right\}.$$

Using Eq. (3.4) for the scale $\tilde{v}_0$, from Eq. (3.5) we determine

$$\alpha_1=-3\varepsilon\,\delta^2\mathrm{Real}\left[\frac{iH_{3/2}(s)H^*_{5/2}(s)}{H^*_{7/2}(s)H_{5/2}(s)}+\frac{1}{2}\frac{H_{3/2}(s)}{H_{7/2}(s)}\right]-\frac{1}{3}\int_1^\infty x^2\Phi_1(x)\,dx.$$

The remaining constants are easily found from the kinematic conditions (3.3). Here we only give the expression for the parameter β_1, which determines the liquid velocity at infinity:

$$\beta_1=-\frac{1}{9}\left[\left.\frac{d\gamma_1}{dr}\right|_{r=1}+2\gamma_1(1)+3\alpha_1\right]. \tag{3.12}$$

4. An analysis of the solution shows that the actual velocity scale of the flow that develops differs from (3.4), and the value $v_0=(3/4)(\delta^2/140)\tilde{v}_0=(3/140)(aH_0^2/8\pi\rho\nu)$ is more suitable. This fact does not invalidate the arguments used to derive (3.4). It is related to the fact that the characteristic size over which the velocity varies from zero (at the surface of the sphere) to its characteristic value does not equal the radius of the sphere, as tacitly assumed in introducing $\tilde{v}_0$, but also depends on the thickness of the skin layer.

The investigated velocity field, as follows from (3.10), is sum of two parts, $\mathbf{V}=\mathbf{V}_0+\varepsilon\mathbf{V}_1$. The first is described by the term (3.10) with a number $l=2$ and does not depend on ε; it corresponds to a central dipole and was investigated in [3]. The presence of the displacement, as noted earlier, can make the sphere self-propelled. It seems that the motion of such a sphere relative to the liquid is directed along the line of action of the force $\mathbf{F}$, i.e., in the direction of the negative z semiaxis. In the coordinate system of the sphere, therefore, the flow velocity at infinity should be directed along z, and its magnitude should be $u_0=-2\beta_1\tilde{v}_0$. Bearing in mind that β_1 of (3.12) is proportional to ε, and using the new velocity scale, we represent the expression for u_0 in the form

$$u_0=\varepsilon\,\frac{3}{140}\,\frac{aH_0^2}{8\pi\rho\nu}\,u(\delta),\qquad u(\delta)=-\frac{2\beta_1}{\varepsilon}\,\frac{140\cdot 4}{3\delta^2}. \tag{4.1}$$

For the case of a strong skin effect, i.e., for $\delta\ll 1$ (see [2]), the constant β_1 of (3.12) can be expanded in an

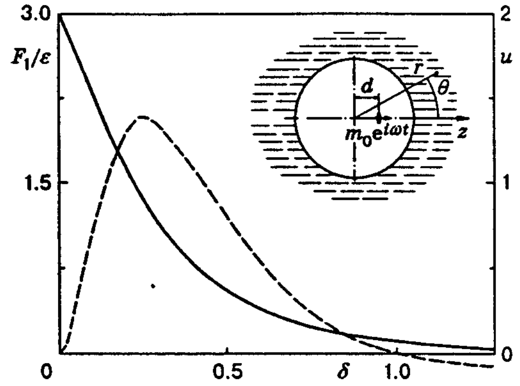

Fig. 1

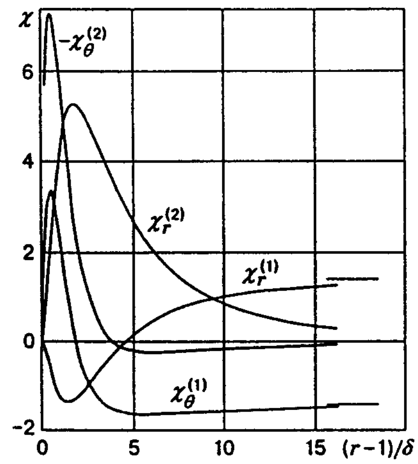

Fig. 2

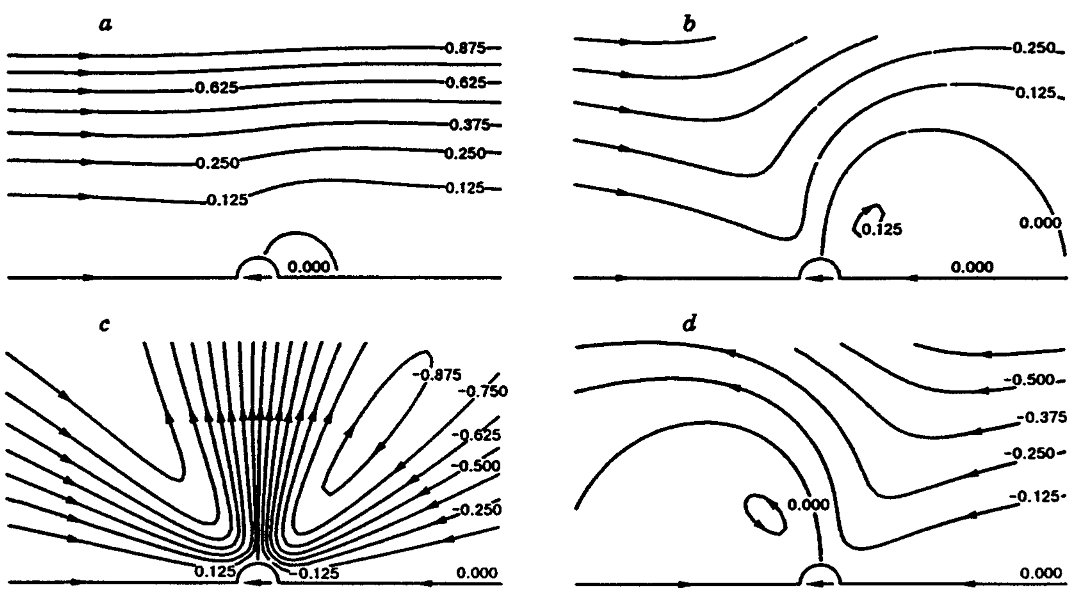

Fig. 3

asymptotic series in powers of δ. The result for $u(\delta)$ has the form

$$u(\delta) = 140\delta^2\Big[1 - \frac{3}{5}\,\delta + O(\delta^2)\Big]. \tag{4.2}$$

The dimensionless thrust (3.9) for $\delta \ll 1$ is

$$F_1(\varepsilon, \delta) = 3\varepsilon\Big[1 - \frac{5}{2}\,\delta + O(\delta^2)\Big]. \tag{4.3}$$

It is seen from (4.1) and (4.2) that for $\delta \ll 1$, the velocity u_0 is proportional to δ^2 (like the velocity $\mathbf{V}_0$ away from a central dipole [3]) and goes to zero as $\delta \to 0$. At first glance, this result seems at variance with Eq. (4.3), according to which the electromagnetic thrust differs from zero even for $\delta = 0$. In fact, there is no contradiction, since for $\delta = 0$ the electromagnetic forces and their curl in the liquid outside the sphere identically vanish. The total force exerted on the sphere also vanishes, since the surface electromagnetic forces lead to a pressure redistribution in the liquid over the surface of the sphere, and the resultant pressure force on the sphere from the liquid balances the electromagnetic thrust. For $\delta = 0$, we are therefore dealing with the strange situation in which, in the absence of motion, the solid body experiences "drag" on the part of the surrounding liquid (pressure drag), which balances the electromagnetic thrust. (As we shall note below, an almost analogous situation arises for $\delta = \delta_* \simeq 1$.) For $\delta > 0$, the thrust exceeds the pressure drag and the sphere goes into motion relative to the liquid.

The dimensionless thrust (3.9) as a function of δ for arbitrary values of δ and the dependence $u(\delta)$ are given in Fig. 1 (solid and dashed curves, respectively). It is seen that the thrust has a constant direction for all δ and decreases with increasing δ. It is seen from the graph of $u(\delta)$ that the maximum dimensionless velocity of the sphere, reached at $\delta \simeq 0.25$, is close to unity. Therefore, εv_0 actually characterizes the scale of the translational velocity of the sphere. As can be seen from Fig. 1, the motion studied has another peculiarity: at $\delta > \delta_* \simeq 1$, the function $u(\delta)$ takes negative values, whereas F_1 is positive. This means that for $\delta > \delta_*$ the sphere moves in the opposite direction from the thrust (the force exerted on the dipole by the magnetic field from currents in the liquid). The possibility of negative velocities can also be seen from the asymptotic behavior (for $\delta \gg 1$) of the expression

$$u(\delta) = -\frac{224}{45}\,\frac{1}{\delta^4} + O\Big(\frac{1}{\delta^5}\Big).$$

To understand the reason for this peculiarity in the motion of the sphere and obtain an idea of the character of the flow around a self-propelled sphere, let us investigate the velocity field, which comprises $\mathbf{V}_0$ and the additional velocity field $\mathbf{V}_1$ proportional to ε. This additional vector field is determined by the first and third terms of (3.11) and can be represented in the form

$$\mathbf{V}_1 = \varepsilon\,\frac{3}{140}\,\frac{aH_0^2}{8\pi\rho\nu}\,\{[\chi_r^{(1)}(r)\cos\theta + \chi_r^{(2)}(r)\cos\theta\sin^2\theta]\mathbf{e}_r + [\chi_\theta^{(1)}(r)\sin\theta + \chi_\theta^{(2)}(r)\sin^3\theta]\mathbf{e}_\theta\},$$

$$\begin{pmatrix} \chi_r^{(1)} \\ \chi_r^{(2)} \\ \chi_\theta^{(1)} \\ \chi_\theta^{(2)} \end{pmatrix} = \frac{4}{3}\,\frac{140}{3\delta^2}\begin{pmatrix} -\dfrac{2}{r}(\psi_1 + 6\psi_3) \\ 30\,\dfrac{\psi_3}{r} \\ \dfrac{1}{r}\,\dfrac{d}{dr}[r(\psi_1 + 6\psi_3)] \\ -\dfrac{15}{2}\,\dfrac{1}{r}\,\dfrac{d}{dr}(r\psi_3) \end{pmatrix}. \tag{4.4}$$

The qualitative behavior of the functions appearing in (4.4) is represented in Fig. 2 by curves for a fixed value of $\delta = 0.25$, which corresponds to the maximum sphere velocity. It is seen that as $r \to \infty$, the functions $\chi_r^{(1)}$ and $\chi_\theta^{(1)}$ asymptotically approach the values $u(\delta)$ and $-u(\delta)$, respectively (horizontal bars), and the functions $\chi_r^{(2)}$ and $\chi_\theta^{(2)}$ approach zero. The absolute values of the functions have sharp maxima near the surface of the sphere, with the maximum values of $|\chi_r^{(2)}|$ and $|\chi_\theta^{(2)}|$ exceeding $|u(\delta)|$ by many times. The flow pattern is therefore characterized by intense vortical flows around the sphere with characteristic velocities considerably exceeding the velocity of translational motion.

This fact means that the surface forces exerted on the sphere by the liquid are determined to a considerable extent by these vortical flows. Hence, for $\delta > \delta_*$, the resultant of these forces can balance the thrust only if the sphere is moving in the z direction. The flow patterns are given in Fig. 3. Streamlines of the total flow, comprising $\mathbf{V}_0$ and $\mathbf{V}_1$, are given for $\varepsilon = 0.1$ and $\delta = 0.25$, 0.8, 1.0, and 1.5 (Fig. 3a–d, respectively). The arrow inside the sphere indicate the direction of the force exerted on the dipole by the magnetic field, and the arrows on the streamlines indicate the direction of the flow velocity (relative to the sphere). It is seen that for all δ for which the translational velocity of the sphere differs from zero, regardless of the direction of motion, the flow is detached. The detached zone lies in the stern region, and its size depends on δ. For $\delta = 1$, the translational velocity of the sphere is almost zero and the flow pattern is reminiscent of that for a central dipole [3]. Recall that for $\delta = 1$, the force $\mathbf{F}$ exerted on the dipole by the magnetic field is balanced by the drag at a zero translational velocity of the sphere, i.e., we have a situation analogous to that described above. Only here, in contrast to the case of $\delta = 0$, the hydrodynamic drag consists of both pressure drag and frictional drag.

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